Statistics

Lecture 8

Feb 19-8:47 AM

SG 16

Binomial Prob. Dist.

1) There are n independent events.

2) Each event has only two outcomes.

$P(\text{Success}) = p \qquad P(\text{Failure}) = q$

$$p + q = 1$$
$$q = 1 - p$$

p & q remain unchanged for all n independent events.

3) $x \Rightarrow$ # of Successes

$n - x \Rightarrow$ # of failures

$$P(x) = \boxed{{}_{n}C_{x}} \cdot p^{x} \cdot q^{n-x}$$

$\Rightarrow$ How many ways we can have x Successes in n trials. order does not matter. No Replacement

We have 10 trials,
How many ways can we have 3 Successes? $n = 10$, $x = 3$

${}_{10}C_{3} = \boxed{120}$ 10 $\boxed{\text{Math}}$ $\boxed{\rightarrow}$ PRB $\boxed{\downarrow}$ $\boxed{{}_{n}C_{r}}$ 3 $\boxed{\text{enter}}$

15 newborn babies, how many ways can we have 4 boys? $n = 15$, $x = 4$

${}_{15}C_{4} = \boxed{1365}$ 15 $\boxed{\text{Math}}$ $\boxed{\rightarrow}$ PRB $\boxed{\downarrow}$ $\boxed{{}_{n}C_{r}}$ 4 $\boxed{\text{enter}}$

Apr 18-8:08 AM

Consider a binomial Prob. dist. with $n=10$ and $p=.6$.

1) $q=1-p=\boxed{.4}$

2) $np=10(.6)=\boxed{6}$

3) $npq=10(.6)(.4)=\boxed{2.4}$

4) $\sqrt{npq}=\sqrt{2.4}\approx\boxed{1.549}$

5) $P(\underbrace{x=7}_{})={}_{10}C_{7}\cdot(.6)^{7}\cdot(.4)^{3}$

7 Successes

$=\boxed{.215}$

$\boxed{{}_{n}C_{x}\cdot p^{x}\cdot q^{n-x}}$

Formula for binomial Prob dist.

Apr 18-8:19 AM

Let's randomly select 16 newborn babies and Success is To have a girl.

1) $n=\boxed{16}$

2) $p=\boxed{.5}$

3) $q=1-p=\boxed{.5}$

4) $np=16(.5)=\boxed{8}$

5) $npq=16(.5)(.5)=\boxed{4}$

6) $\sqrt{npq}=\sqrt{4}=\boxed{2}$

7) $P(\underbrace{\text{exactly } 10}_{x=10}\text{ girls})=P(x=10)$

$n-x=16-10=6$

${}_{n}C_{x}\cdot p^{x}\cdot q^{n-x}$

$={}_{16}C_{10}\cdot(.5)^{10}\cdot(.5)^{6}$

$=\boxed{.122}$

USING TI Command

[2nd] [VARS] [↓] [binompdf](16, .5, 10) [Enter]

Trials: 16

P: .5

x-Value: 10

[Paste] [Enter]

Your work:

$\boxed{\begin{array}{l}P(x=10)= \\ \text{binompdf}(16,.5,10)= \\ .122\end{array}}$

Apr 18-8:28 AM

You are making random guesses on an exam with 100 True/False questions.

Success is to guess correct answer.

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $np = 100(.5) = 50$ 5) $npq = 100(.5)(.5) = 25$ 6) $\sqrt{npq} = \sqrt{25} = 5$

7) P(guess correctly on 60 answers)

$= P(x = 60) = \text{binompdf}(\overset{n}{100}, \overset{p}{.5}, \overset{x}{60}) = \boxed{.011}$

8) P(guess Correctly on at most 60 answers)

$= P(x \le 60) = \text{binomcdf}(100, .5, 60) = \boxed{.982}$

Apr 18-8:39 AM

You are making random guesses on a multiple-choice exam with 40 questions.

Each question has 5 choices but only one correct choice.

Success is to guess correct answer.

1) $n = 40$ 2) $p = \frac{1}{5} = .2$ 3) $q = \frac{4}{5} = .8$

4) $np = 40(.2) = \boxed{8}$ 5) $npq = 40(.2)(.8) = \boxed{6.4}$ 6) $\sqrt{npq} = \sqrt{6.4}$ Round to a whole # $= 2.530 \approx \boxed{3}$

7) P(guess exactly 10 Correct Ans.)

$P(x = 10) = \text{binompdf}(40, .2, 10) = \boxed{.107}$

8) P(guess at most 10 Correct Ans.)

$P(x \le 10) = \text{binomcdf}(40, .2, 10) = \boxed{.839}$

9) P(guess fewer than 10 Correct Ans.)

$P(x < 10) = P(x \le 9) = \text{binomcdf}(40, .2, 9) = \boxed{.732}$

Apr 18-8:50 AM

A loaded Coin is tossed 80 times.
Success is to land tails.
Prob. of landing tails on each toss is $\frac{1}{4}$.

1) $n = 80$ 2) $p = .25$ 3) $q = .75$

4) $np = 80(.25) = \boxed{20}$

5) $npq = 80(.25)(.75) = \boxed{15}$

6) $\sqrt{npq} = \sqrt{15} \approx \boxed{4}$

7) P(lands exactly 30 tails)

$P(x = 30) = \text{binompdf}(80, .25, 30) = \boxed{.004}$

8) P(lands fewer than 30 tails)

$P(x < 30) = P(x \leq 29) = \text{binomcdf}(80, .25, 29) = \boxed{.991}$

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9) P(lands tails at least 15 times)

$P(x \geq 15) = 1 - P(x \leq 14)$

Total Prob.

We don't Want — 14 | 15 — We want

$= 1 - \text{binomcdf}(80, .25, 14) = \boxed{.926}$

10) P(# of tails is between 15 and 25, inclusive)

$P(15 \leq x \leq 25) = P(x \leq 25) - P(x \leq 14)$

Reduce by 1

$= \text{binomcdf}(80, .25, 25) - \text{binomcdf}(80, .25, 14)$

14 15 25

$= \boxed{.846}$

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FedEx Says 90% of all deliveries are ontime.
60 packages were randomly selected.

Success is for delivery not to be late.

1) $n = 60$ 2) $p = .9$ 3) $q = .1$

4) $np = 60(.9) = 54$

5) $npq = 60(.9)(.1) = \boxed{5.4}$

6) $\sqrt{npq} = \sqrt{5.4} \approx 2.324 \approx \boxed{2}$

7) P(at most 55 are ontime)

$P(x \le 55) = \text{binomcdf}(60, .9, 55) = \boxed{.729}$

8) P(at least 50 are on time)

$P(x \ge 50) = 1 - P(x \le 49)$

we want

49 50

$= 1 - \text{binomcdf}(60, .9, 49)$

$= \boxed{.966}$

10) P(# of ontime deliveries are between 52 and 58, inclusive).

$P(52 \le x \le 58) = P(x \le 58) - P(x \le 51)$

51 52 58

$= \text{binomcdf}(60, .9, 58) - \text{binomcdf}(60, .9, 51)$

$= \boxed{.845}$

Apr 18-9:34 AM

Mean $\boxed{\mu = np}$

Variance $\boxed{\sigma^2 = npq}$

Standard Deviation $\boxed{\sigma = \sqrt{\sigma^2}}$

Binomial Prob. Dist.

Consider a binomial Prob. dist with $n = 400$ & $P = .5$

1) $q = 1 - P = \boxed{.5}$

2) $\mu = np = 400(.5) = \boxed{200}$

3) $\sigma^2 = npq = 400(.5)(.5) = \boxed{100}$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{100} = \boxed{10}$

5) 68% Range $\Rightarrow \mu \pm \sigma = 200 \pm 10 \rightarrow \boxed{190 \text{ to } 210}$

6) $\underbrace{\text{Usual Range}}_{\text{95\% Range}} \Rightarrow \mu \pm 2\sigma = 200 \pm 2(10) \rightarrow \boxed{180 \text{ to } 220}$

7) $P(180 \le x \le 220) = \text{binomcdf}(400, .5, 220) - \text{binomcdf}(400, .5, 179) = \boxed{.960}$

Reduce by 1

$\approx 96\%$

Apr 18-9:48 AM

Prob. dist. with Continuous Random Variable

SG 17-20

1) Uniform Prob. dist.

2) Standard Normal Prob. dist.

3) Normal Prob. dist.

4) Central Limit Theorem (CLT)

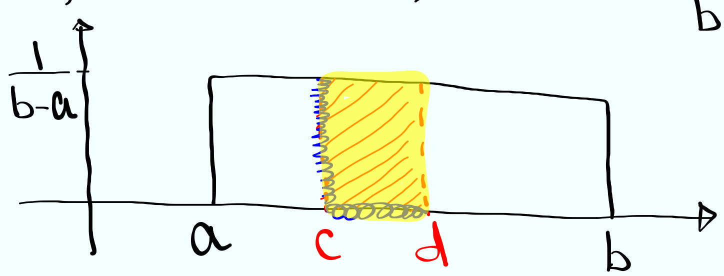
5) Applications

Apr 18-10:13 AM

Uniform Prob. dist.

1) Graph is rectangular with total area $= 1$.

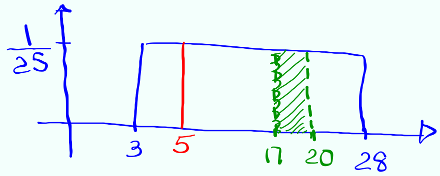
2) Length $a \le x \le b$, width $\frac{1}{b-a}$



3) $P(c < x < d) = (d-c)\cdot \frac{1}{b-a}$ 4) $P(x=c)=\boxed{0}$

Apr 18-10:17 AM

Consider a Uniform Prob. dist for all Values from 3 to 28.



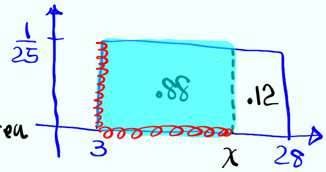
1) $P(x=5)=0$
Line Area

2) $P(17<x<20)=(20-17)\cdot\frac{1}{25}$
$=\boxed{\frac{3}{25}}$

3) Find $x=P_{88}$

88% below
Left Area
.88

12% above
Right Area
.12



$(x-3)\cdot\frac{1}{25}=.88$

Multiply by 25

$x-3=25(.88)$

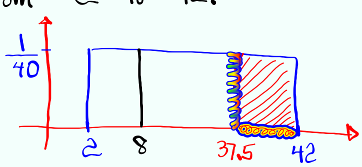
$x-3=22$

$x=22+3$

$\boxed{x=25}$

Apr 18-10:21 AM

Consider a uniform Prob. dist. for all Values from 2 to 42.

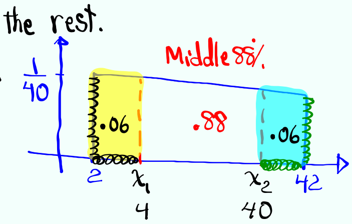


1) $P(x=8)=0$
Line

2) $P(x>37.5)=(42-37.5)\cdot\frac{1}{40}=\frac{4.5}{40}=\frac{9}{80}$
$=.1125\approx.113$

3) Find two Values that Separate the middle 88% from the rest.

Round to whole #.



$(x_1-2)\cdot\frac{1}{40}=.06$

$x_1-2=40(.06)$

$x_1=2.4+2=4.4\approx\boxed{4}$

$(42-x_2)\cdot\frac{1}{40}=.06$

$42-x_2=40(.06)$

$42-x_2=2.4$

$42-2.4=x_2$

$x_2=39.6\approx\boxed{40}$

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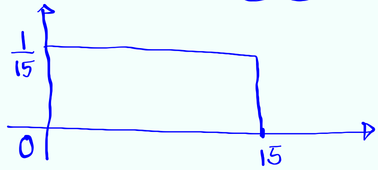
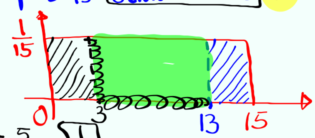
Quiz time has a uniform prob. dist of 15 mins.



1) P(time taken for the quiz is $\boxed{\text{below 3 mins}}$ or $\boxed{\text{above 13 mins}}$)

$= 1 - P(3 < x < 13)$



$= 1 - (13-3)\cdot\frac{1}{15} = 1 - \frac{10}{15} = \frac{5}{15} = \boxed{\frac{1}{3}}$

2) P(time taken for the quiz is below 3 mins and above 13 mins) $= \boxed{0}$

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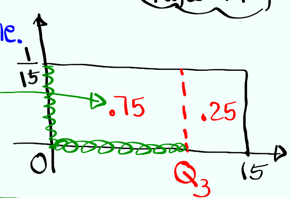
3) Find Q_3 for quiz time.



$(Q_3 - 0)\cdot\frac{1}{15} = .75$

$Q_3 - 0 = 15(.75)$

$Q_3 = 11.25 \approx \boxed{11 \text{ mins}}$

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